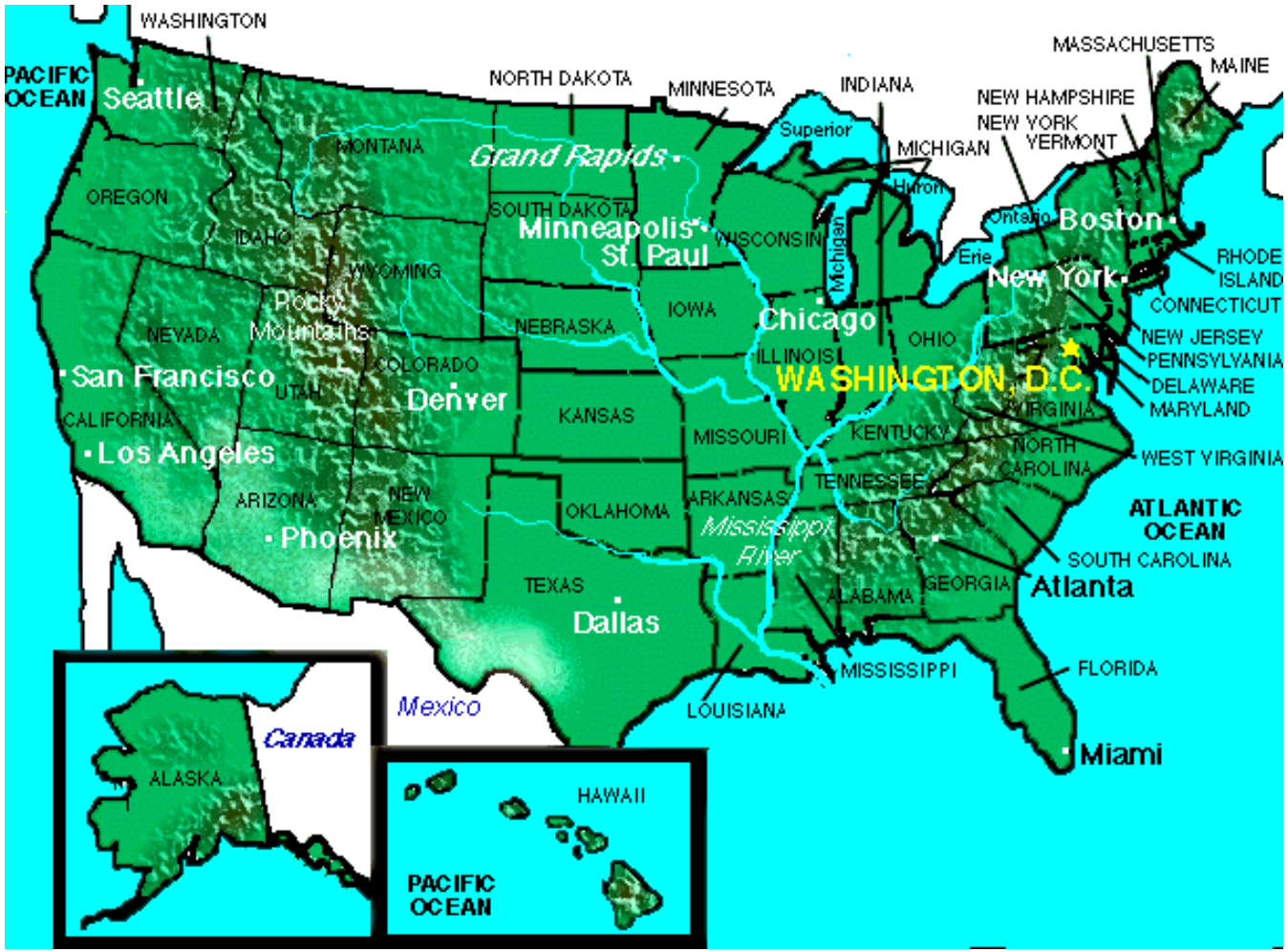
Example: Map coloring



Dynamic backtracking

- $expl_{ik}$
  - explanation for *eliminating* the value v_k for variable x_i
  - set of previously assigned variables that imply that x_i cannot take the value v_k
  - analogous to a combination of mbl_{ik} and $conf\text{-}set_i$ generalized to non-binary constraints
  - can be set by forward or backward checking
- *assigned*
  - sequence of assigned variables
- *unassigned*
  - set of unassigned variables

Dynamic backtrack: *label*

function *dbt-label*(*i*)

update-explanations(*i*)

if $CD_i \neq \{\}$ **then**

Select some $v_k \in CD_i$ and set $x_i = v_k$

push(*i*, *assigned*) and *unassigned* = *unassigned* \ {*i*}

return (*NextVar*(), *true*)

else

return (*i*, *false*)

end *dbt-label*

Updating eliminating explanations

procedure *update-explanations*(i)

    for each $v_k \in CD_i$ **do**

        for each completed constraint C that refers to x_i **do**

            if C is not satisfied **then**

                $expl_{ik} = vars(C) \setminus \{x_i\}$

                $CD_i = CD_i \setminus \{v_k\}$

                break inner loop

            endif

end *update-explanations*

Dynamic backtracking: *unlabel*

function *dbt-unlabel*(*i*)

$expl = \cup_r expl_{ir}$

if $expl = \{\}$ **then return** (*0*, *false*)

Let $j \in expl$ be the *last* element pushed onto *assigned* and

let v_k be currently assigned to x_j

for each l assigned *after* j and each value $v_m \in D_l$ **do**

if $j \in expl_{lm}$ **then**

$expl_{lm} = \{\}$

$CD_l = CD_l \cup \{v_m\}$

endif

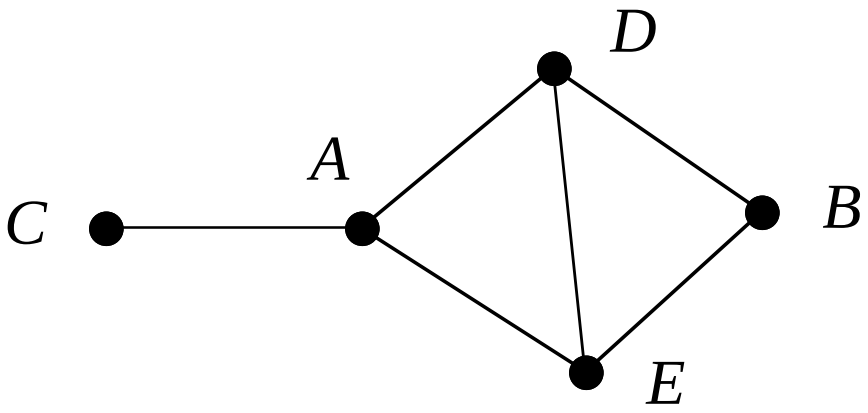
$assigned = assigned \setminus \{j\}$ and $unassigned = unassigned \cup \{j\}$

$expl_{jk} = expl \setminus \{j\}$

return (*NextVar*(), *true*)

end *dbt-unlabel*

Example



Country	*color*	*red*	*yellow*	*blue*
A				
B				
C				
D				
E				

Termination of dynamic backtracking

- Setting $expl_{ik} = \{y_1,\ldots, y_l\}$ is equivalent to asserting the nogood

$$y_1 = d_1,\ldots, y_l = d_l \Rightarrow x_i \neq v_k$$

- Let $\{s_1,\ldots,s_m\}$ be the variables that *precede* x_i when $expl_{ik}$ is set

$\Rightarrow$ the following *modified nogood* is entailed

$$s_1 = e_1,\ldots, s_m = e_m \Rightarrow x_i \neq v_k$$

- **Lemma:** As long as the modified nogood is entailed by some nogood, the algorithm will not visit the node

$$s_1 = e_1,\ldots, s_m = e_m,\ x_i = v_k$$

- **Corollary:** Whenever a nogood is asserted, a previously unentailed modified nogood is entailed

Termination (contd.)

- **Lemma:** When the algorithm deletes a nogood, the corresponding modified nogood continues to be entailed
- **Corollary:** The set of entailed modified nogoods grows monotonically

- **Theorem:** Dynamic backtracking terminates